Finite State Machines

Outline and Objectives

Outline

- Motivation
- Example: Door Controller
- Formal Definition
- Example/Activity: Vending Machine

Learning Objectives

- Describe finite state machine (FSM).
- Give an example of a task or action that can be modelled by a FSM.
- Convert a simple FSM to an implementation in sequential logic.

Motivation

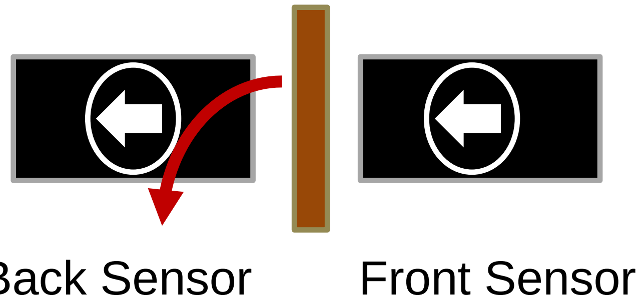
Observation: When we act, we

- look at our current situation (take into account our current “state” and environment/inputs)
- respond (“act”) and enter a new situation

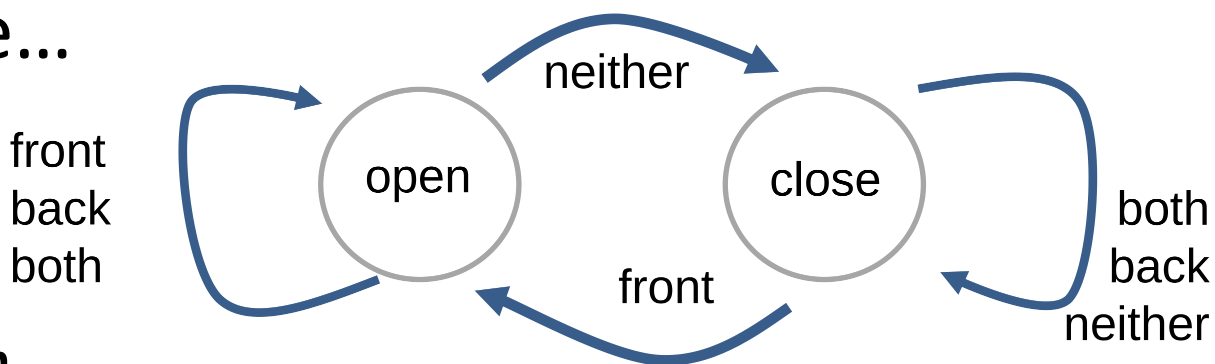
This same behavior can be modelled by simple “machines”.

Example: Door Controller

Consider a controller that opens an automatic door…



We would like to regulate when the door should open and close…

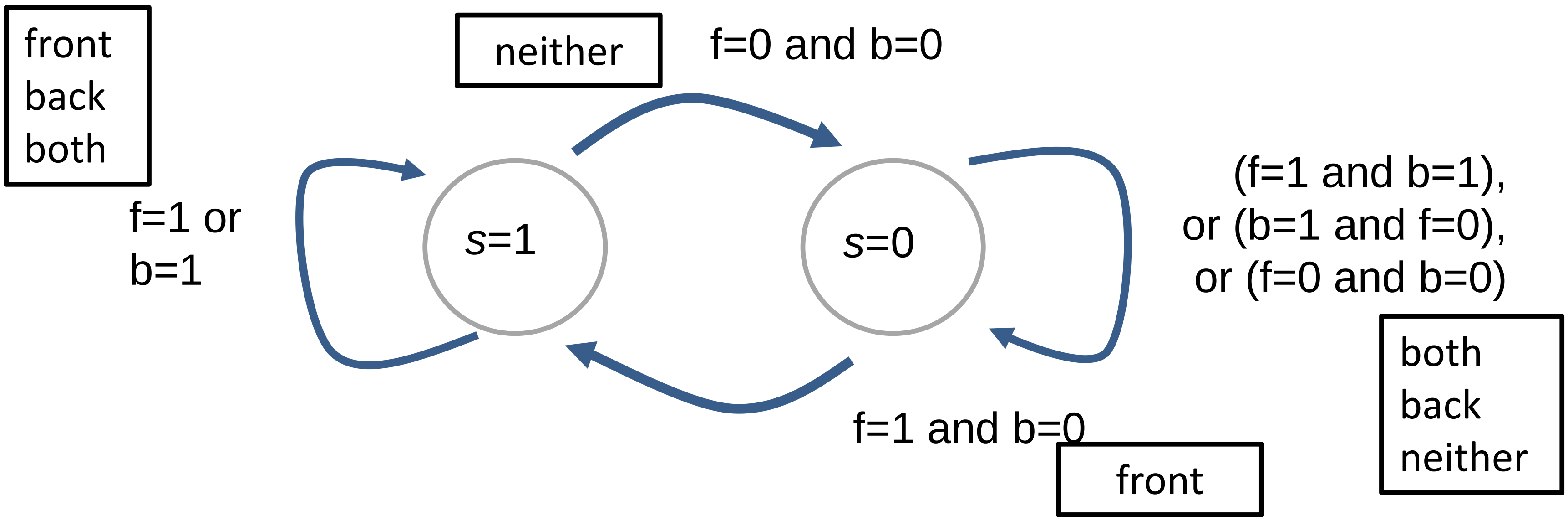


State Diagram

Example: Door Controller

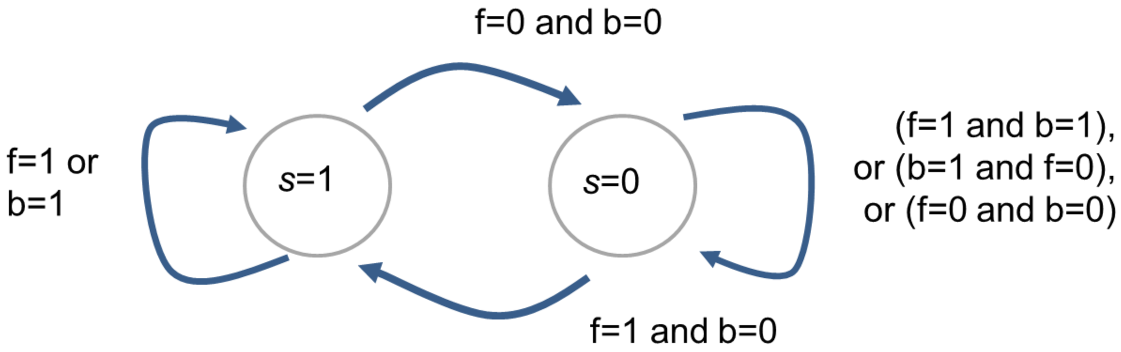
Now we need to convert this diagram into something we can implement with a circuit…

- encode the states in Boolean variables
- represent the sensors as Boolean variables



Example: Door Controller

Create a truth table taking three inputs (f, b, and s) and one output (s^+)



f	*b*	*s*	$s^+(f,b,s)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Example: Door Controller

We will use a Kmap to simply the formula for s^+

f	*b*	*s*	$s^+(f,b,s)$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

fb/s	**00**	**01**	**11**	**10**
0	0	0	0	1
1	0	1	1	1

$$s^+(f, b, s) = s\,b + f\,\overline{b}$$

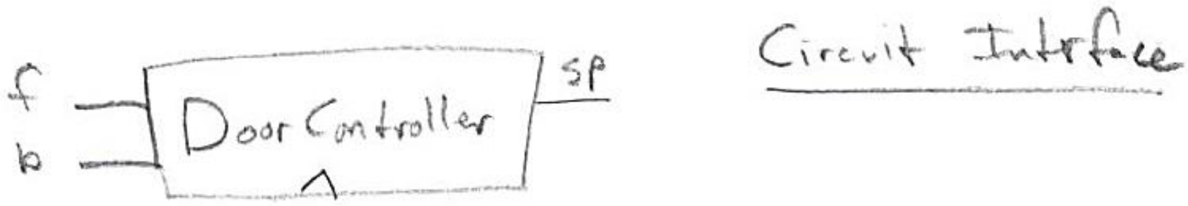
Example: Door Controller

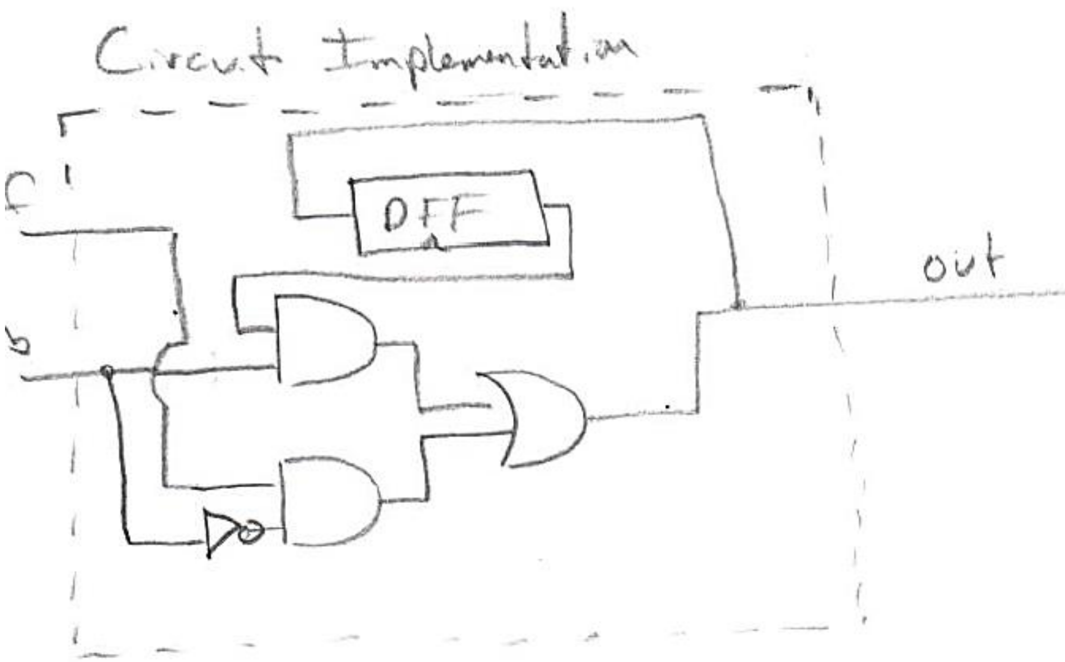
$s^{+}(f, b, s) = s\,b + f\,\overline{b}$

Note how the output (next state) depends on the current state.

This means we will need what type of logic to implement the controller?

Example: Door Controller





Formal Definition

A FSM can be fully specified by

- Finite set of states *S*
- A start state q_0 ε *S*
- A set of accept states *F*
- A finite set of valid inputs *X* (i.e., domain of input)
- A transition function, *f*, that maps current state and input to the next state

(Sipser, 35)

Example: Vending Machine

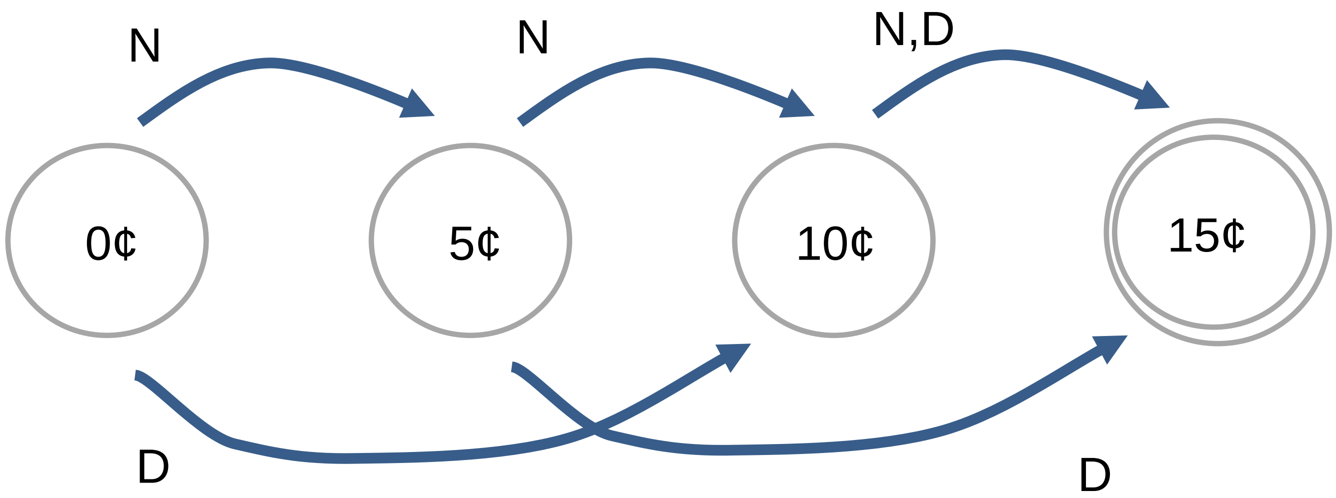
We need to construct a simple machine that will determine if a combination of nickels and dimes should dispense a gumball.

We should dispense the gumball when we've received 15¢ (i.e., accept when we have the $$).

(Katz, 389)

Example: Vending Machine

We create a state diagram to see how we could arrive at an accept state.



Example: Vending Machine

Formalizing a FSM

- S = {0¢, 5¢, 10¢, 15¢}
- q_0 = 0¢
- F = {15¢}
- X = {N, D}

f(X,S)

Curr. State	Coin Rcvd.	Next State
0¢	N	5¢
0¢	D	10¢
5¢	N	10¢
5¢	D	15¢
10¢	N	15¢
10¢	D	15¢

Example: Vending Machine

Activity: Convert this FSM to a circuit.

First: Determine an encoding for the states and create Boolean expressions for the transition function

Then: Convert the Boolean expression to a logic diagram and implement in HDL.

References

- Sipser M: *Introduction to the theory of computation*. 2nd ed. Boston: Thomson Course Technology; 2006.
- Katz RH: *Contemporary logic design*. Redwood City, Calif: Benjamin/Cummings; 1994.